Study Concerning Some Elasticity Characteristics Determination of Composite Bars

ION CIUCĂ[1]*, DUMITRU BOLCU[2], MARIUS MARINEL STĂNESCU[3], GHEORGHE MARIN[1], SMARANDA DANIELA IONESCU[4]

[1] University Politehnica of Bucharest, 313 Splaiul Independenţei, 060042, Bucharest , Romania

[2] University of Craiova,165 Calea Bucureşti Str., 200620, Craiova, Romania

[3] University of Craiova, 13 A.I.Cuza Str., 200396, Craiova, Romania

[4] Oradea University, Textiles and Leather, 1 Universităţii Str., 410087, Oradea, Romania

In this paper the authors propose a new method to determine the properties of a composite material made from two different components: fiber glass cloth and resin. On the separation surface between the two materials it is assumed that the continuity conditions concerning the tensors of stresses and strains between these two components are totally satisfied. The constitutive equations were determined for each material as functions of these components with respect to a local system of reference. Medium stresses and strains were defined in order to establish the properties of the material as a whole. Considering the interdependence existing at the level of those properties the constitutive equations of the composite have been found. Mathematical expressions for elastic modulus and Poisson's ratio have been also determined. A loading with forces applied on a longitudinal as well on a perpendicular direction face to the fibers direction of the composite is considered. We find out that the elastic properties along the fibers of the composite are closer to the properties of the reinforcing fibers and the elastic properties perpendicular to the fibers of the composite are closer to the matrix properties.

Keywords: composite, elastic properties, constitutive equations

The composite materials allow us to obtain a diversity of mechanical properties but it turns out to be really difficult to determine the mechanical characteristics as function of fibers/matrix ratio. All methods of investigation of the properties and behaviours of the composite materials require the knowledge of a large number of elastic constants, so, the models are very complex [1]. The properties of composites depend on certain issues like:

- mechanical and elastical properties of the constituents;
- volumetric fraction of the constituents;
- geometric arrangement of the constituents;
- adherence between the materials;
- the fibers length;
- manufacturing process.

We can distinguish three types of methods used to analyze the properties of composite materials:

- finding the extreme values using variational energy theorems;
- finding the exact solution;
- semi-empirical approximations.

Lower and upper limits for elasticity modules of composites having fibers with various diameters and randomly disposed with a given volumetric proportion were found [2,3]. Similar studies for composites having hexagonally disposed fibers with constant diameter were made [2,4].

Some studies in order to obtain the exact solutions considering the case of a square elementary cell in which the section of the reinforcing component has a square form were made [5, 6]. Although this kind of cell isn't common in practice, it has the advantage that allows a mathematical transcription of the fiber-matrix interaction taking into account simultaneously the differences between the arrangements of the fibers over two different directions of the section composite. Based on these results was proposed a model that allows the calculus of the shear modulus [7].

A micro-mechanics analysis for the substances of the composite allowed the determination of the composite behaviour using the known properties of each individual constituent and the particularities of interaction between any two different constituents [7]. This method has the advantage of generating the hypotheses during analysis. The result is a macro-mechanical evaluation for the behaviour of the composite, considering the heterogeneous compound as being a homogenous transversally isotropic one, for which we can find the properties taking into consideration several fiber-matrix combinations.

Based on theoretically-obtained results it was proposed the use of some simple relations which give good results for Young's modulus and Poisson's ratio along the fibers [8]. The other elastic coefficients can be only empirically determined because they depend on a parameter characterizing the fiber-matrix interaction, fiber geometry and arrangement.

Some numerical results for a periodic arrangement of fibers, using a finite differences method were obtained [9], these numerical results corresponding very well to the semi-empirical results obtained for cylindrical fibers in volumetric proportion of approximately 0,5 [8].

The properties of composite materials with spherical inclusions using the finite element method were analyzed [10].

The same finite element method was used and the properties of a composite bar having long fibers with a double periodicity in their arrangement over the section of the bar was determined [11]. Moreover, it was made a probabilistic analysis for the properties of unidirectional composites [12].

In [13-14] we find an analysis of the viscoelastic behaviour of technical polymers and advanced materials with epoxy resin.

The previous theoretical results have been obtained both for unidirectional composite as well as for multilayer composites having a different fiber orientation for each layer.

For composite having the reinforcing component built as a two or three- dimensional fabric or randomly disposed

* *email: ion.ciuca@mec.edu.ro*

we found only semi-empirical results as a conclusion of a statistical analysis [15].

Theoretical considerations. The determination of the constitutive equations conecting the stress and strains tensors

Since only some of the theories used to determine composite materials' characteristics have a solid theoretical background, the obtained theoretical results correspond more or less to those experimentally obtained. Briefly, a tensile stress will lead to an elongation following the stress direction and an uniform compression can not lead to an expansion/dilatation of the material. We can not give credibility to any result contradicting even partially these elementary truths.

The authors considered a composite bar made from two different materials, so, the bar is basically heterogeneous. We note with 1 the fibers material and with 2 the matrix material. All calculus is related to a global reference system having x_3 axis along the bar, so, the x_3 axis is perpendicular to the axis x_1 and x_2.

Considering a local reference system for each and every point of the bar placed on the separation surface between materials. This reference system has its axis in such way that:

- n axis perpendicular to the separation surface between materials;
- t axis tangent to the separation surface from the bar's section;
- τ axis tangent to the separation surface, parallel with the bar's longitudinal axis.

Under external stress, in the composite for each material will appear:

- a stress state:

$$\left\{\sigma_n^{(i)}\right\} = \\ = \left\{\sigma_{\tau\tau}^{(i)};\sigma_{tt}^{(i)};\sqrt{2}\sigma_{\tau t}^{(i)};\sqrt{2}\sigma_{nt}^{(i)};\sqrt{2}\sigma_{n\tau}^{(i)};\sigma_{nn}^{(i)}\right\}^t, \quad (1)$$
$$i = 1,2;$$

-a strain state:

$$\left\{\varepsilon_n^{(i)}\right\} = \\ = \left\{\varepsilon_{\tau\tau}^{(i)};\varepsilon_{tt}^{(i)};\sqrt{2}\varepsilon_{\tau t}^{(i)};\sqrt{2}\varepsilon_{nt}^{(i)};\sqrt{2}\varepsilon_{n\tau}^{(i)};\varepsilon_{nn}^{(i)}\right\}^t, \quad (2)$$
$$i = 1,2.$$

The next conditions must be fulfilled on the separation surface between materials:

- continuity conditions for stresses:

$$\sigma_{nn}^{(1)} = \sigma_{nn}^{(2)} = \sigma_{nn};\sigma_{nt}^{(1)} = \sigma_{nt}^{(2)} = \sigma_{nt}; \\ \sigma_{n\tau}^{(1)} = \sigma_{n\tau}^{(2)} = \sigma_{n\tau}; \quad (3)$$

- continuity conditions for strains:

$$\varepsilon_{tt}^{(1)} = \varepsilon_{tt}^{(2)} = \varepsilon_{tt};\varepsilon_{\tau\tau}^{(1)} = \varepsilon_{\tau\tau}^{(2)} = \varepsilon_{\tau\tau}; \\ \varepsilon_{t\tau}^{(1)} = \varepsilon_{t\tau}^{(2)} = \varepsilon_{t\tau}. \quad (4)$$

We note:

$$\left\{\varepsilon_n^{(i)}\right\} = \begin{Bmatrix}\{\varepsilon^*\}\\ \{\varepsilon_{ni}\}\end{Bmatrix};\left\{\sigma_n^{(i)}\right\} = \begin{Bmatrix}\{\sigma_{ni}\}\\ \{\sigma^*\}\end{Bmatrix}, i = 1,2; \quad (5)$$

where:

$$\{\varepsilon^*\} = \begin{Bmatrix}\varepsilon_{\tau\tau}^{(1)}\\ \varepsilon_{tt}^{(1)}\\ \sqrt{2}\varepsilon_{\tau t}^{(1)}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_{\tau\tau}^{(2)}\\ \varepsilon_{tt}^{(2)}\\ \sqrt{2}\varepsilon_{\tau t}^{(2)}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_{\tau\tau}\\ \varepsilon_{tt}\\ \sqrt{2}\varepsilon_{\tau t}\end{Bmatrix}; \quad (6)$$

$$\{\varepsilon_{ni}\} = \begin{Bmatrix}\sqrt{2}\varepsilon_{nt}^{(i)}\\ \sqrt{2}\varepsilon_{n\tau}^{(i)}\\ \varepsilon_{nn}^{(i)}\end{Bmatrix}, i = 1,2; \quad (7)$$

$$\{\sigma^*\} = \begin{Bmatrix}\sqrt{2}\sigma_{nt}^{(1)}\\ \sqrt{2}\sigma_{n\tau}^{(1)}\\ \sigma_{nn}^{(1)}\end{Bmatrix} = \begin{Bmatrix}\sqrt{2}\sigma_{nt}^{(2)}\\ \sqrt{2}\sigma_{n\tau}^{(2)}\\ \sigma_{nn}^{(2)}\end{Bmatrix} = \begin{Bmatrix}\sqrt{2}\sigma_{nt}\\ \sqrt{2}\sigma_{n\tau}\\ \sigma_{nn}\end{Bmatrix}; \quad (8)$$

$$\{\sigma_{ni}\} = \begin{Bmatrix}\sigma_{\tau\tau}^{(i)}\\ \sigma_{tt}^{(i)}\\ \sqrt{2}\sigma_{\tau t}^{(i)}\end{Bmatrix}, i = 1,2. \quad (9)$$

For $\{\varepsilon^*\}$ and $\{\sigma^*\}$ given by (6) and (8) there is no need to use the index pointing to the constituent so we are omitting it. Since the stresses and strains must fulfill the continuity conditions on the separation surface between materials, these tensors can be described by continuous functions not depending on material or the area each material separately occupies.

So, the constitutive equations connecting the stresses and strains tensors have the form:

$$\begin{Bmatrix}\{\varepsilon^*\}\\ \{\varepsilon_{ni}\}\end{Bmatrix} = \begin{bmatrix}[A_i] & [B_i]\\ [B_i]^t & [C_i]\end{bmatrix}\begin{Bmatrix}\{\sigma_{ni}\}\\ \{\sigma^*\}\end{Bmatrix}, \quad (10)$$

where:

$$[A_i] = \begin{bmatrix}\frac{1}{E_i} & -\frac{\upsilon_i}{E_i} & 0\\ -\frac{\upsilon_i}{E_i} & \frac{1}{E_i} & 0\\ 0 & 0 & \frac{1}{G_i}\end{bmatrix}, [B_i] = \begin{bmatrix}0 & 0 & -\frac{\upsilon_i}{E_i}\\ 0 & 0 & -\frac{\upsilon_i}{E_i}\\ 0 & 0 & 0\end{bmatrix},$$

$$[C_i] = \begin{bmatrix}\frac{1}{G_i} & 0 & 0\\ 0 & \frac{1}{G_i} & 0\\ 0 & 0 & \frac{1}{G_i}\end{bmatrix}, i = 1,2; \quad (11)$$

and E_i is Young's modulus for the material "i"; G_i is the sliding modulus for the material "i"; ν_i Poisson's ratio for "i" material.

From (10) we obtain:

$$\{\sigma_{ni}\} = [A_i]^{-1}\{\varepsilon^*\} - \\ -[A_i]^{-1}[B_i]\{\sigma^*\}, i = 1,2; \quad (12)$$

$$\{\varepsilon_{ni}\} = [B_i]^t[A_i]^{-1}\{\varepsilon^*\} + \\ +\left([C_i] - [B_i]^t[A_i]^{-1}[B_i]\right)\{\sigma^*\}, i = 1,2. \quad (13)$$

So, the strain and stresses matrices can be expressed with respect to the components fulfilling the continuity conditions (3) and (4):

$$\left\{\varepsilon_n^{(i)}\right\}=\begin{bmatrix}[I] & [0]\\ [D_i]' & [L_i]\end{bmatrix}\begin{Bmatrix}\{\varepsilon^*\}\\ \{\sigma^*\}\end{Bmatrix}, i=1,2; \tag{14}$$

$$\left\{\sigma_n^{(i)}\right\}=\begin{bmatrix}[T_i] & -[D_i]\\ [0] & [I]\end{bmatrix}\begin{Bmatrix}\{\varepsilon^*\}\\ \{\sigma^*\}\end{Bmatrix}, i=1,2; \tag{15}$$

where:

$$[T_i]=\begin{bmatrix}\frac{E_i}{1-\upsilon_i^2} & \frac{E_i\upsilon_i}{1-\upsilon_i^2} & 0\\ \frac{E_i\upsilon_i}{1-\upsilon_i^2} & \frac{E_i}{1-\upsilon_i^2} & 0\\ 0 & 0 & G_i\end{bmatrix}; [D_i]=\begin{bmatrix}0 & 0 & -\frac{\upsilon_i}{1-\upsilon_i}\\ 0 & 0 & -\frac{\upsilon_i}{1-\upsilon_i}\\ 0 & 0 & 0\end{bmatrix};$$

$$[L_i]=\begin{bmatrix}\frac{1}{G_i} & 0 & 0\\ 0 & \frac{1}{G_i} & 0\\ 0 & 0 & \frac{1-\upsilon_i-2\upsilon_i^2}{E_i(1-\upsilon_i)}\end{bmatrix}; i=1,2, \tag{16}$$

[I], [0] are the unit matrix and null matrix respectively.

The elastic characteristics of the bar with respect to the global reference system and are presented above:

$$\left\{\varepsilon^{(i)}\right\}=\begin{bmatrix}[M(\theta)] & [N(\theta)]\\ [N(-\theta)]' & [P(\theta)]\end{bmatrix}\cdot \begin{bmatrix}[I] & [0]\\ [D_i]' & [L_i]\end{bmatrix}\cdot\begin{Bmatrix}\{\varepsilon^*\}\\ \{\sigma^*\}\end{Bmatrix}; \tag{17}$$

$$\left\{\sigma^{(i)}\right\}=\begin{bmatrix}[M(\theta)] & [N(\theta)]\\ [N(-\theta)]' & [P(\theta)]\end{bmatrix}\cdot \begin{bmatrix}[T_i] & -[D_i]\\ [0] & [I]\end{bmatrix}\cdot\begin{Bmatrix}\{\varepsilon^*\}\\ \{\sigma^*\}\end{Bmatrix}; \tag{18}$$

where:

$$[M(\theta)]=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos^2\theta & 0\\ 0 & 0 & \cos\theta\end{bmatrix};$$

$$[N(\theta)]=\begin{bmatrix}0 & 0 & 0\\ -\sqrt{2}\cos\theta\sin\theta & 0 & \sin^2\theta\\ 0 & -\sin\theta & 0\end{bmatrix};$$

$$[P(\theta)]= \tag{19}$$

$$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & 0 & -\sqrt{2}\cos\theta\sin\theta\\ 0 & \cos\theta & 0\\ \sqrt{2}\cos\theta\sin\theta & 0 & \cos^2\theta\end{bmatrix},$$

θ is the angle between n and x_1 axes.

We make the following hypothesis: all the constituents of the composite are placed the way that the whole composite structure should have not a relative sliding on the common boundaries between constituents. This means a perfect adherence between constituents. In these conditions we may consider the composite as a continuous material medium, so, we have only one constitutive equation. This way, a homogenization of the composite material is done.

We define:

- matrix of medium deformations:

$$\{\bar{\varepsilon}\}=\frac{1}{S}\sum_{i=1}^{2}\iint_{S_i}\left\{\varepsilon^{(i)}\right\}dS; \tag{20}$$

- matrix of medium tensions:

$$\{\bar{\sigma}\}=\frac{1}{S}\sum_{i=1}^{2}\iint_{S_i}\left\{\sigma^{(i)}\right\}dS; \tag{21}$$

where S is the surface of the bar's transversal section and S_i is the surface of the material "i" as part of the transversal section.

According to the previous conditions, the constitutive equation of composite has the form:

$$\{\bar{\sigma}\}=[E]\{\bar{\varepsilon}\}, \tag{22}$$

or

$$\{\bar{\varepsilon}\}=[C]\{\bar{\sigma}\}, \tag{23}$$

where $[E]$ and $[C]$ are the stiffness and flexibility matrixes of the entire composite.

The authors concluded that the global elastic properties of the composite bar depend also on the elastic properties of the constituents as well as on their geometric distribution into the section.

For composites reinforced with long fibers we can identify an elementary repetitive cell across the section. That's why we can calculate the integrals from (20) and (21) on the elementary cell and after repeating this calculus for all the section we will obtain the geometrical distribution of the components.

The classic theories consider the reinforcement and matrix as being elastically parallel mediums.

So, based on the constitutive equation, [17, 18] we can determine the elastic coefficients as follows:

- elasticity modulus transversally to the fibers

$$\frac{1}{E_T}=\frac{2\left[\upsilon_1V_1(1-\upsilon_2)+\upsilon_2V_2(1-\upsilon_1)\right]^2}{(1-\upsilon_1)(1-\upsilon_2)}\cdot \frac{\left[V_1E_1(1+\upsilon_2)+V_2E_2(1+\upsilon_1)\right]}{\left[V_1^2E_1^2\left(1-\upsilon_2^2\right)+V_2^2E_2^2\left(1-\upsilon_1^2\right)+2V_1V_2E_1E_2\left(1-\upsilon_1\upsilon_2\right)\right]}+ +\frac{V_1(1+\upsilon_1)(1-2\upsilon_1)}{(1-\upsilon_1)E_1}+\frac{V_2(1+\upsilon_2)(1-2\upsilon_2)}{(1-\upsilon_2)E_2}; \tag{24}$$

- elasticity modulus along the fibers

$$E_L= =\frac{V_1^2E_1^2\left(1-\upsilon_2^2\right)+V_2^2E_2^2\left(1-\upsilon_1^2\right)+2V_1V_2E_1E_2\left(1-\upsilon_1\upsilon_2\right)}{V_1E_1\left(1-\upsilon_2^2\right)+V_2E_2\left(1-\upsilon_1^2\right)}; \tag{25}$$

- Poisson's ratios coefficients

$$\upsilon_{TT} = \frac{V_1\upsilon_1 E_1\left(1-\upsilon_2^2\right)+V_2\upsilon_2 E_2\left(1-\upsilon_1^2\right)}{V_1 E_1\left(1-\upsilon_2^2\right)+V_2 E_2\left(1-\upsilon_1^2\right)}, \quad (26)$$

$$\upsilon_{LT} = \frac{\left[\upsilon_1 V_1\left(1-\upsilon_2\right)+\upsilon_2 V_2\left(1-\upsilon_1\right)\right]}{V_1 E_1\left(1-\upsilon_2^2\right)+V_2 E_2\left(1-\upsilon_1^2\right)} \cdot \quad (27)$$

$$\cdot\left[V_1 E_1\left(1+\upsilon_2\right)+V_2 E_2\left(1+\upsilon_1\right)\right],$$

where V_1 is the reinforcement volume fraction, V_2 is the matrix volume fraction, the form of them being:

$$V_1 = \frac{S_1}{S}; V_2 = \frac{S_2}{S}. \quad (28)$$

Experimental researches

We consider a composite bar built in epoxy resin reinforced with long fibers glass. The two components have the following elastic components:

- *for the epoxy resin*

1)Young elasticity modulus E = 4500 MPa;
2)shearing modulus G = 1600 MP;
3)transversal contraction coefficient (Poisson's ratio) ν = 0.4;

- *for fiber glass wires*

1) Young modulus E = 74000 MPa;
2) shearing modulus G= 30000 MPa;
3) transversal contraction coefficient (Poisson's ratio) ν = 0.25.

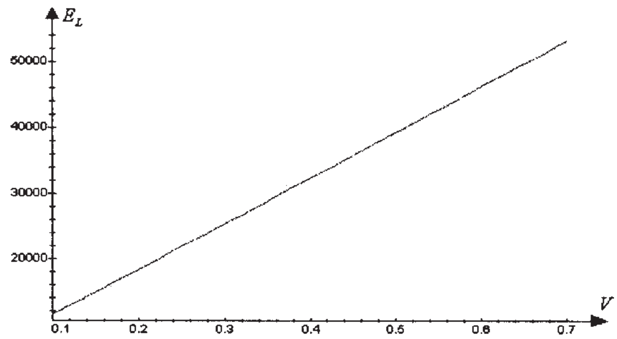

Fig. 1. The variation of Young's modulus along the fibers in the reference system V, E_L

In figure 1 we present the variation of the elasticity modulus along the fibers, variation noted as E_L and expressed as a function of the reinforcing component volumetric ratio. The composite Young's modulus of rises proportionally with the reinforcement's mass.

In figure 2 we present the variation of the elasticity modulus over a direction perpendicular to the fibers, variation noted with E_T. That has a relatively slow evolution with respect to the volume fraction of the reinforcing component having smaller values than E_L. The explanation consists in the fact that we can find portions with only epoxy resin on a direction perpendicular on the fibers.

In figure 3 we presented the variation of the transversal contraction coefficient (Poisson's ratio) ν_{LT}, expressed as a function of the volume fraction of the reinforcing component. This variation is characteristic for the transversal strain due to a longitudinal unitary stress and tends very fast to the value of the Poisson's ratio of fibers. We can assume that, beginning with the volume fraction of the fibers V=0.3, the transversal contraction coefficient of the composite is very close to the transversal contraction coefficient of the reinforcing component.

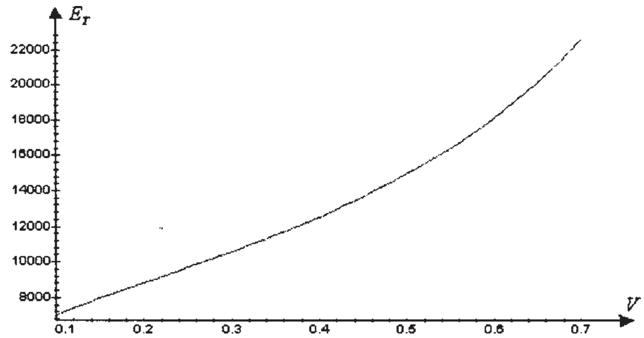

Fig. 2 The transversal variation of Young's modulus with respect to the reference system V, E_T

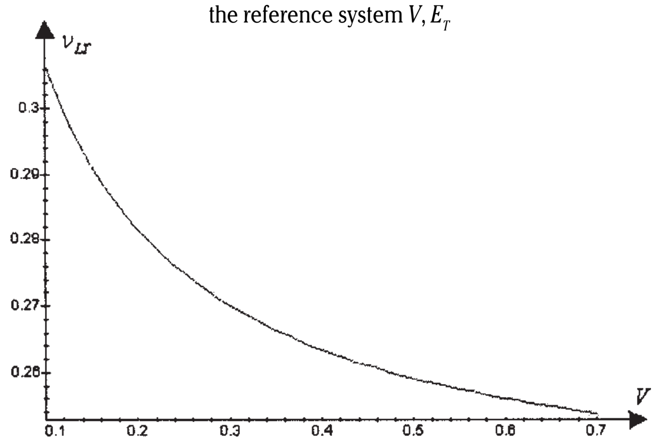

Fig. 3. The variation of the transversal contraction coefficient (Poisson's ratio) ν_{LT} as function of the volumetric ratio of the reinforcing component with respect to the reference system V, ν_{LT}

In figure 4 we present the variation of the transversal contraction coefficient (Poisson's ratio) ν_{TT} which characterizes the transversal deformation due to a unitary stress perpendicular on the fibers. The coefficient stays near by the value of the Poisson's ratio for the matrix but diminishing as function of the volume fraction of the reinforcing component. For comparison, for a volume fraction of reinforcing component V = 0.5 we have ν_{LT}= 0.259 and ν_{TT}=0.333.

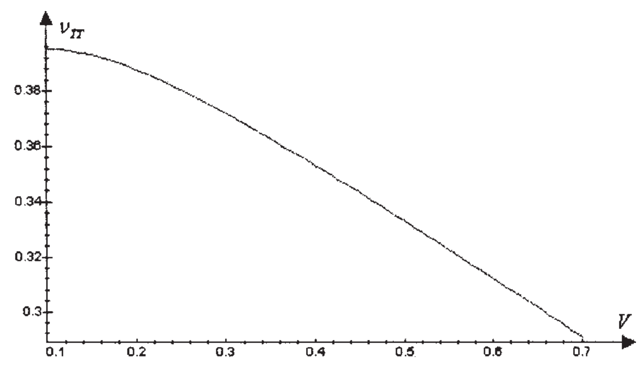

Fig. 4. The variation of transversal contraction coefficient (Poisson's ratio) ν_{TT} as function of the volumetric ratio of reinforcing component with respect to the reference system V, ν_{TT}

The most important elastic constant of a material is the elasticity modulus.For its determination, a device was used, presenting the loading scheme from figure 5, where the test boards were fixed.

The shape and the dimensions of the test boards are presented in figure 6.

Successive charges, were applied and the displacement was measured using a comparative device .

For the calculation of the elasticity modulus the following formula was used:

$$E = \frac{L_0^3}{4bh^3} \cdot \frac{\Delta P}{\Delta f} [MPa], \qquad (29)$$

where

L_o = the distance between the supports [mm];
b = the width of the test board [mm] ;
h = the thickness of the test board [mm];
ΔP = the variation of the external loading [N];
Δf = the variation of the displacement [mm].

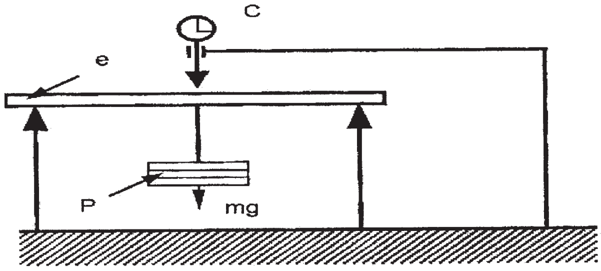

Fig. 5. Experimental setup: (e) - test board; (C) - comparative device; (P) - external loading

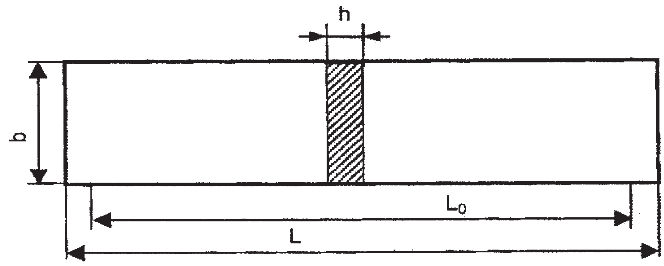

Fig. 6. Shape and dimessions of the test boards

Measurements were made for test boards made of resin (E = 4500, ν = 0.4) reinforced with glass fibres (E = 74000, ν = 0.25), for three volume fraction of the reinforcing component. The results are presented in table 1.

The most used formula for the calculus of the elasticity modulus for a biphasic composite material is (see for example [15, 16]):

$$E_L = E_1V_1 + E_2V_2. \qquad (30)$$

Table 1
EXPERIMENTAL MODULUS OF ELASTICITY

No.	Volume ratio		Dimensions of the test board			Experimental data		Modulus of elasticity $E[MPa]$
	Glass	Resin	L_0 $[mm]$	b $[mm]$	h $[mm]$	ΔP $[N]$	Δf $[mm]$	
1	0.3	0.7	210	25.1	2.6	0.98	0.21	25268
						1.96	0.40	
						2.94	0.61	
2	0.4	0.6	210	25.5	2.8	0.98	0.12	33022
						1.96	0.25	
						2.94	0.37	
3	0.5	0.5	210	25.8	3.1	1.96	0.15	39149
						2.94	0.23	
						3.92	0.30	

Table 2
COMPARATIVELY RESULTS OF THE MODULUS OF ELASTICITY

No.	Volume ratio		Modulus of elasticity $E[MPa]$		
	V_1	V_2	Relation (29)	Relation (35)	Experimental results
1	0.3	0.7	25422	25350	25268
2	0.4	0.6	33394	32300	33022
3	0.5	0.5	39306	39250	39149

In table 2 are comparatively presented the results obtained through the relations (25) and (30) and the experimental ones.

If the coefficients of Poisson's ration have the same value, for both constituents, then the relations (25) and (30) are identical.

If the constituent materials have Poisson's ratio differ very much one from another, then just the formula (25) will supply us a result close by of reality.

Conclusions

We notice that along the fibers the elastic properties of the composite are similar to the elastic properties of the reinforcement while perpendicular on the fibers the elastic properties of the composite are close to matrix properties.

Beside these specific conclusions we can make some general conclusions according to the elastic properties of the composite depending on:

- elastic properties of the constituents;
- volumetric proportion of the constituents;
- constituents' distribution across the section;

Also the composite reinforced with long fibers behaves like a homogeneous material having transversal isotropy.

The authors consider that the use of these results and formulas is suitable in the calculus of structures made of composite materials, where it is compulsory to appreciate the mechanical behavior based on the properties of the homogeneous materials used. Because the properties of the composites depend on fibers orientation one could optimize the use of the properties of the material if the fibers are arranged along the maximum stress direction.

Acknowledgement: For Stănescu M.M. and Ciuca I. this work was partially financed from the Romanian Ministry of Education, Research and Youth (A.N.C.S. and A.M.C.S.I.T), grant PN-CDI – II, No. 1262/2007; authors are expressing their thanks to the participants to Applied Mechanics Seminar from Department of Mechanic, University of Craiova, for their remarks and helpful comments.

References

1. BOLCU, D., STĂNESCU, M., CIUCĂ, I., JIGA, G., GHEORGHIU, H., IANCULUI, D:, Mat. Plast., **45**, nr. 2, 2008, p. 137
2. HASKIN, Z., The elastic model of heterogeneous materials, ASME J. Appl. Mech., 29, 1962, p. 143
3. HILL, R., A self-consistent mechanics of composite materials, Mech. Phys. Solids, 13, 1965, p. 213
4. HASKIN, Z., ROSEN, B.W., Fibers composites analysis and design, **I**, Composite Materials and Laminates, Material Sciences Corporation, 1983
5. ABOUDI, J., Micromechanical analysis of composites by the method of Cells, Applied Mechanics Reviews, 42, 1989, p. 193
6. CEDERBAUM, G., ELISHAKOFF, I., ABOUDI, J., LIBRESCU, L., Random vibration and reliability of composite structures, Technomic Publishing Company, Inc., Lancaster, Pennsylvania, 1992
7. CHRISTENSEN, R., ZYWICS, E., Dimensional constitutive theory for liber composite laminated media, J. Appl., Mech., **57**, nr. 4, 1990, p. 948
8. HALPIN, J.C., ASHTON, J.E., PETIT, P.H., Primer on composite materials: analysis, Technomic Publ. Co., Westport, 1969
9. AHMAD, J., A micromechanics analysis of cracks in unidirectional fiber composites, J. Appl. Mech., 58, 1991, p. 964
10.HUANG, Y., HU, K.X., Elastic moduli of microcracked composite with spherical inclusions of cubic anisotropy, Composite Science and Technology, 50, nr. 2, 1994, p. 149
11.MEGUID, S.A., KALAMKAROV, A.L., ZOUGAS, A., Analysis of local and effective elastic properties of fibre-reinforced materials, Compas. Eng., 4, nr. 5, 1994, p. 525
12.PYRZ, R., Quantitative description of the microstructure of composites, Part I: Morphology of unidirectional, Composites Science and Technology, 50, nr. 2, 1994, p. 197
13.CASCAVAL, C-TIN., ROSU, D., ROSU, L., Mat. Plast., **36**, nr. 1, 2005, p. 76
14.PUSCA, ST., PAUN, M.A., TOMA, C., Mat. Plast., **44**, nr . 1, 2007, p. 39
15. BERTHELOT, J.M., Composite materials-mechanical behavior and structure analysis, Mason Publishers, 1992
16. HADĂR, A., BORDEAŞU, I., MITELEA, I., VLĂSCEANU, D., Mat. Plast., **43**, nr. 1, 2006, p. 70
17. PAPANICOLAU, G.C., JIGA, G., GIANNIS, S.P., Mat. Plast., **44**, nr. 3, 2007, p. 250
18. HADĂR, A:, JIGA, G., BAYER, M., Mat. Plast., 42, nr. 2, 2005, p. 100

Manuscript received: 14.11.2007